

Grade: XII

MATHEMATICS –Sample paper-1

Time: 3 hours

Marks:100

SECTION-A

Answer the following:

4x1=4

1. Let * be a binary operation on the set of all non zero real numbers given by $a * b = \frac{ab}{5}$ for all $a, b \in R - \{0\}$. Find the value of x , given that $2*(x*5)=10$.
2. Evaluate: $\int \cos^{-1}(\sin x) dx$.
3. Write the differential equation representing the family of lines $y = mx$, where m is an arbitrary constant.
4. Find $|\vec{x}|$, if for a unit vector \vec{a} , $(\vec{x} - \vec{a})(\vec{x} + \vec{a}) = 15$.

SECTION-B

5. If $\sin\left(\sin^{-1} \frac{1}{5} + \cos^{-1} x\right) = 1$, then find the value of x . **8 x 2 = 16**

6. Using elementary transformations, find the inverse of $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

7. If $\vec{a} = i + j + k, \vec{b} = j - k$, find a vector c , such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 0$

8. The equations of a line are $5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.

9. Write the Cartesian equation of a plane, bisecting the line segment joining the points $(2, 3, 5)$ and $(4, 5, 7)$ at right angles.

10. Evaluate: $\int \frac{2}{1 + \cos 2x} dx$

11. The volume of a sphere is increasing at the rate of 3 cubic centimetre per second. Find the rate of increase of its surface area, when the radius is 2cm.

12. Show that the function $f(x) = 2x - |x|$ is continuous but not differentiable at $x = 0$.

SECTION-C

Answer the following:

11 x 4 = 44

13. Show that $f : N \rightarrow N$, given by $f(x) = \begin{cases} x+1, & x \text{ is odd} \\ x-1, & x \text{ is even} \end{cases}$ is both one-one and onto.

14. Solve for x : $\cos(\tan^{-1} x) = \sin\left(\cot^{-1} \frac{3}{4}\right)$. (OR)
 If $2 \tan^{-1}(\cos \theta) = \tan^{-1}(2 \operatorname{cosec} \theta)$, then find the value of θ .
15. If A is a square matrix such that $A^2 = A$, then write the value of $7A - (I + A)^3$, where I is an identity matrix.
16. Find the value of a for which the function f defined as

$$f(x) = \begin{cases} a \sin \frac{\pi}{2}(x+1), & \text{if } x \leq 0 \\ \frac{\tan x - \sin x}{x^3} & \text{if } x > 0 \end{cases}$$
 is continuous at $x = 0$.
17. Find the points on the curve $9y^2 = x^3$, where the normal to the curve makes equal intercepts on the axes.
18. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1 + e^x} dx$
19. Using integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.
20. Form the differential equation of the family of circles in the second quadrant and touching the coordinate axes.
21. Show that the vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if and only if $\vec{a} + \vec{b}, \vec{b} + \vec{c}, \vec{c} + \vec{a}$ are coplanar.
22. Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line whose direction cosines are proportional to $2, 3, -6$.
23. Three machines A, B, C in a certain factory producing electric bulbs, produce 50%, 25% and 25% respectively, of the total daily output of electric bulbs. It is known that 4% of the bulbs produced by each of machines A and B are defective and that 5% of those produced by machine C are defective. If one bulb is picked up at random from a day's production, calculate the probability that it is defective.

SECTION-D

Answer the following:

6 x 6 = 36

24. Show that the relation S in the set R of real numbers, defined as $S = \{(a, b) : a, b \in R \text{ and } a \leq b^3\}$ is neither reflexive, nor symmetric, nor transitive.

25. Prove: $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$
26. Prove that the semi-vertical angle of the right circular cone of the given volume and least curved surface area is $\cot^{-1} \sqrt{2}$.
27. Using integration find the area of the following region:
 $\{(x, y) : |x-1| \leq y \leq \sqrt{5-x^2}\}$
28. An aeroplane can carry a maximum of 200 passengers. A profit of Rs.500 is made on each executive class ticket out of which 20% will go to the welfare fund of the employees. Similarly a profit of Rs.400 is made on each economy ticket out of which 25% will go for the improvement of facilities provided to economy class passengers. In both cases, the remaining profit goes to the airline's fund. The airline reserves at least 20 seats for executive class. However at least four times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the net profit of the airline? Make the above as LPP and solve graphically. Do you think, more passengers would prefer to travel by such an airline than by others.
29. Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II at random. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.